## Cambridge IGCSE ${ }^{\text {TM }}$



## CAMBRIDGE INTERNATIONAL MATHEMATICS

Paper 5 Investigation (Core)
May/June 2022
1 hour 10 minutes
You must answer on the question paper.
No additional materials are needed.

## INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.


## INFORMATION

- The total mark for this paper is 36 .
- The number of marks for each question or part question is shown in brackets [ ].

This document has 12 pages. Any blank pages are indicated.

Answer all the questions.

## INVESTIGATION

## CIRCLES AND REGIONS

This is an investigation into the number of regions formed by drawing lines on a circle.

## 1 Radii

The diagrams show the number of regions inside a circle when 1 radius and 2 radii are drawn. The regions inside the circle are numbered.


1 radius


2 radii
(a) (i) Draw radii on the circles below and number the regions.


3 radii


4 radii


5 radii
(ii) Complete the table.

| Number of <br> radii | Number of regions |
| :---: | :---: |
| 1 | 1 |
| 2 | 2 |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |

(b) Write a formula, in terms of $n$, for the number of regions, $R$, when there are $n$ radii.
$\qquad$

## 2 Diameters

The diagrams show the number of regions inside a circle when 1 diameter and 2 diameters are drawn.


1 diameter


2 diameters
(a) Complete the table for 3, 4 and 5 diameters.

You may use the empty circle to help you.

| Number of <br> diameters | Number of regions |
| :---: | :---: |
| 1 | 2 |
| 2 | 4 |
| 3 |  |
| 4 |  |
| 5 |  |


(b) Write a formula, in terms of $n$, for the number of regions, $R$, when there are $n$ diameters.

## 3 Chords

In this investigation:

- each chord must cut every other chord
- only two chords may intersect at any point.

The diagrams show the number of regions inside a circle when 1 chord, 2 chords and 3 chords are drawn.


1 chord


2 chords


3 chords
(a) Count the number of regions in the circle when 4 chords are drawn.

(b) Complete this table.

You may use the empty circle to help you.

| Number of <br> chords | Number of <br> regions |
| :---: | :---: |
| 1 | 2 |
| 2 | 4 |
| 3 | 7 |
| 4 |  |
| 5 |  |
| 6 | 22 |


[2]
(c) This is a formula for the number of regions, $R$, when there are $n$ intersecting chords.

$$
R=\frac{1}{2} n^{2}+b n+1
$$

Find the value of $b$.

## 4 Tangents

A region can be inside or outside the circle when the lines are tangents.
These two diagrams both show a circle with 2 tangents and the regions numbered.
The maximum number of regions for a circle with 2 tangents is 6 .

(a) Give a reason why the first diagram does not have the maximum number of regions with 2 tangents.
$\qquad$
$\qquad$
(b) (i) Use this diagram to find the maximum number of regions when there are 3 tangents.

(ii) Draw a fourth tangent on the diagram below to find the maximum number of regions.

(c) Use your answers to part (b) to complete the table.

| Number of <br> tangents | Maximum <br> number of regions |
| :---: | :---: |
| 1 | 3 |
| 2 | 6 |
| 3 |  |
| 4 |  |
| 5 |  |

(d) This is a formula for the maximum number of regions, $R$, when there are $n$ tangents.

$$
R=\frac{1}{2} n^{2}+b n+1
$$

Find the value of $b$.

## 5 Secants

A secant is a straight line that intersects a circle at two points and extends outside the circle.
In this investigation:

- each secant must cut every other secant
- only 2 secants may intersect at any point
- secants must not intersect on the circumference of the circle.


The diagrams show the number of regions when 1 secant and 2 secants are drawn.


1 secant

(a) Draw a third secant on the diagram below to find the number of regions when there are 3 secants. Complete the table.


| Number of <br> secants | Number of <br> regions |
| :---: | :---: |
| 1 | 4 |
| 2 | 8 |
| 3 |  |
| 4 | 19 |
| 5 | 26 |

(b) This is a formula for the number of regions, $R$, when there are $n$ secants.

$$
R=\frac{1}{2} n^{2}+b n+c
$$

Find the value of $b$ and the value of $c$.

$$
\begin{aligned}
& b= \\
& c=
\end{aligned}
$$

6 There are two circles.
The first circle has chords drawn on it.
The second circle has tangents drawn on it.
The number of chords on the first circle is the same as the number of tangents on the second circle.
Each circle has the maximum number of regions.
One circle has 60 more regions than the other.
(a) Find the number of straight lines on each diagram.
(b) Find the larger number of regions.

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